Limit theorems for a general stochastic rumour model

Pablo Martín Rodríguez[1]

Institute of Mathematics, Statistics and Scientific Computation
University of Campinas

Joint work with E. Lebensztayn and F. P. Machado

(Institute of Mathematics and Statistics - University of São Paulo)

7 ERPEM

Santa Fe, December 2nd 2010

[1] Supported by FAPESP (10/06967-2)

The classical stochastic rumour models

The two classical models for the spreading of a rumour are:

1. the Daley-Kendall model (DK model) introduced in 1965;
2. the Maki-Thompson model (MT model) introduced in 1973.

In both models a closed homogeneously mixing population of $N+1$ individuals is subdivided into three classes:

- ignorants: individuals who are ignorant of the rumour;
- spreaders: individuals who are actively spreading the rumour;
- stiflers: individuals who know the rumour but have ceased spreading it.

Remark

Both models consider initially 1 *spreader and* N *ignorants.*

The classical stochastic rumour models

The two classical models for the spreading of a rumour are:

1. the Daley-Kendall model (DK model) introduced in 1965;
2. the Maki-Thompson model (MT model) introduced in 1973.

In both models a closed homogeneously mixing population of $N + 1$ individuals is subdivided into three classes:

- ignorants: individuals who are ignorant of the rumour;
- spreaders: individuals who are actively spreading the rumour;
- stiflers: individuals who know the rumour but have ceased spreading it.

Remark

Both models consider initially 1 *spreader and* N *ignorants.*

The classical stochastic rumour models

The two classical models for the spreading of a rumour are:

1. the Daley-Kendall model (DK model) introduced in 1965;
2. the Maki-Thompson model (MT model) introduced in 1973.

In both models a closed homogeneously mixing population of $N+1$ individuals is subdivided into three classes:

- ignorants: individuals who are ignorant of the rumour;
- spreaders: individuals who are actively spreading the rumour;
- stiflers: individuals who know the rumour but have ceased spreading it.

Remark

Both models consider initially 1 spreader and N ignorants.

The classical stochastic rumour models

The two classical models for the spreading of a rumour are:

1. the Daley-Kendall model (DK model) introduced in 1965;
2. the Maki-Thompson model (MT model) introduced in 1973.

In both models a closed homogeneously mixing population of $N+1$ individuals is subdivided into three classes:

- ignorants: individuals who are ignorant of the rumour;
- spreaders: individuals who are actively spreading the rumour;
- stiflers: individuals who know the rumour but have ceased spreading it.

Remark

Both models consider initially 1 *spreader and* N *ignorants.*

The MT model

In the MT model the rumour is propagated through the population by directed contact between spreaders and other individuals. Then

- when a spreader interacts with an ignorant, the ignorant becomes a spreader;
- whenever a spreader contacts a stifler, the spreader turns into a stifler;
- when a spreader meets another spreader, the initiating one becomes a stifler.

In the MT model the rumour is propagated through the population by directed contact between spreaders and other individuals. Then

- when a spreader interacts with an ignorant, the ignorant becomes a spreader;
- whenever a spreader contacts a stifler, the spreader turns into a stifler;
- when a spreader meets another spreader, the initiating one becomes a stifler.

The MT model

In the MT model the rumour is propagated through the population by directed contact between spreaders and other individuals. Then

- when a spreader interacts with an ignorant, the ignorant becomes a spreader;
- whenever a spreader contacts a stifler, the spreader turns into a stifler;
- when a spreader meets another spreader, the initiating one becomes a stifler.

The MT model

In the MT model the rumour is propagated through the population by directed contact between spreaders and other individuals. Then

- when a spreader interacts with an ignorant, the ignorant becomes a spreader;
- whenever a spreader contacts a stifler, the spreader turns into a stifler;
- when a spreader meets another spreader, the initiating one becomes a stifler.

The MT model

We denote,

- by $X(t)$ the number of *ignorants* at time t,
- by $Y(t)$ the number of *spreaders* at time t,
- by $Z(t)$ the number of *stiflers* at time t.

Then, the process $\{(X(t), Y(t))\}_{t\geq 0}$ is a CTMC with transitions and corresponding rates given by

transition	rate
$(-1, 1)$	XY,
$(0, -1)$	$(N - X)Y$.

We denote,

- by $X(t)$ the number of *ignorants* at time t,
- by $Y(t)$ the number of *spreaders* at time t,
- by $Z(t)$ the number of *stiflers* at time t.

Then, the process $\{(X(t), Y(t))\}_{t\geq 0}$ is a CTMC with transitions and corresponding rates given by

transition	rate
$(-1,1)$	XY,
$(0,-1)$	$(N-X)Y$.

The DK model

In the DK model people interact by pairwise contacts and when two spreaders meet both become stiflers.

In this case the process $\{(X(t), Y(t))\}_{t\geq 0}$ is a CTMC with transitions and corresponding rates given by

transition	rate
$(-1, 1)$	$XY,$
$(0, -2)$	$\binom{Y}{2},$
$(0, -1)$	$(N+1-X-Y)Y.$

Remark

The main subject under study is the proportion of ignorants at the end of the process.

The DK model

In the DK model people interact by pairwise contacts and when two spreaders meet both become stiflers.

In this case the process $\{(X(t), Y(t))\}_{t\geq 0}$ is a CTMC with transitions and corresponding rates given by

transition	rate
$(-1, 1)$	$XY,$
$(0, -2)$	$\binom{Y}{2},$
$(0, -1)$	$(N+1-X-Y)Y.$

Remark

The main subject under study is the proportion of ignorants at the end of the process.

The DK model

In the DK model people interact by pairwise contacts and when two spreaders meet both become stiflers.

In this case the process $\{(X(t), Y(t))\}_{t\geq 0}$ is a CTMC with transitions and corresponding rates given by

transition	rate
$(-1, 1)$	$XY,$
$(0, -2)$	$\binom{Y}{2},$
$(0, -1)$	$(N + 1 - X - Y)Y.$

Remark

The main subject under study is the proportion of ignorants at the end of the process.

Some references

- Sudbury (1985): proves that this proportion converges in probability to ≈ 0.203 (MT model)
- Watson (1988): generalizes the last result using normal asymptotic approximation (MT and DK model)
- Daley and Gani (1999): analyses the (α, p) version of the DK model
- other variations are introduced by Pearce (2000), Hayes (2005) and Kawachi (2008).

Our contribution

1. *we introduce the (α, p, q) version for the DK model;*
2. *we obtain limit theorems for a general rumour model that has as particular cases our generalization and the other mentioned models.*

Some references

- Sudbury (1985): proves that this proportion converges in probability to ≈ 0.203 (MT model)
- Watson (1988): generalizes the last result using normal asymptotic approximation (MT and DK model)
- Daley and Gani (1999): analyses the (α, p) version of the DK model
- other variations are introduced by Pearce (2000), Hayes (2005) and Kawachi (2008).

Our contribution

1. *we introduce the (α, p, q) version for the DK model;*
2. *we obtain limit theorems for a general rumour model that has as particular cases our generalization and the other mentioned models.*

Some references

- Sudbury (1985): proves that this proportion converges in probability to ≈ 0.203 (MT model)
- Watson (1988): generalizes the last result using normal asymptotic approximation (MT and DK model)
- Daley and Gani (1999): analyses the (α, p) version of the DK model
- other variations are introduced by Pearce (2000), Hayes (2005) and Kawachi (2008).

Our contribution

1. *we introduce the (α, p, q) version for the DK model;*
2. *we obtain limit theorems for a general rumour model that has as particular cases our generalization and the other mentioned models.*

Some references

- Sudbury (1985): proves that this proportion converges in probability to ≈ 0.203 (MT model)
- Watson (1988): generalizes the last result using normal asymptotic approximation (MT and DK model)
- Daley and Gani (1999): analyses the (α, p) version of the DK model
- other variations are introduced by Pearce (2000), Hayes (2005) and Kawachi (2008).

Our contribution

1. *we introduce the (α, p, q) version for the DK model;*
2. *we obtain limit theorems for a general rumour model that has as particular cases our generalization and the other mentioned models.*

Some references

- Sudbury (1985): proves that this proportion converges in probability to ≈ 0.203 (MT model)
- Watson (1988): generalizes the last result using normal asymptotic approximation (MT and DK model)
- Daley and Gani (1999): analyses the (α, p) version of the DK model
- other variations are introduced by Pearce (2000), Hayes (2005) and Kawachi (2008).

Our contribution

1. *we introduce the (α, p, q) version for the DK model;*
2. *we obtain limit theorems for a general rumour model that has as particular cases our generalization and the other mentioned models.*

Some references

- Sudbury (1985): proves that this proportion converges in probability to ≈ 0.203 (MT model)
- Watson (1988): generalizes the last result using normal asymptotic approximation (MT and DK model)
- Daley and Gani (1999): analyses the (α, p) version of the DK model
- other variations are introduced by Pearce (2000), Hayes (2005) and Kawachi (2008).

Our contribution

1. *we introduce the (α, p, q) version for the DK model;*
2. *we obtain limit theorems for a general rumour model that has as particular cases our generalization and the other mentioned models.*

The (α, p, q)-DK model

Let $\alpha, p, q \in (0,1]$ and suppose that, independently,

- a spreader involved in a meeting decides to tell the rumour with probability p;
- once such a decision is made, any spreader in a meeting with somebody informed has probability α of becoming a stifler;
- upon hearing the rumour, an ignorant becomes a spreader or a neutral with resp. probabilities q and $1-q$.

Then, if $U(t)$ is the number of neutrals at time t, the CTMC $(X(t), U(t), Y(t))$ evolves according to

transition	rate
$(-1,0,1)$	$pq\, XY,$
$(-1,1,0)$	$p(1-q)\, XY,$
$(0,0,-2)$	$\alpha^2 p(2-p)\binom{Y}{2},$
$(0,0,-1)$	$\alpha(1-\alpha)p(2-p)\, Y(Y-1) + \alpha p\, Y(N+1-X-Y).$

The (α, p, q)-DK model

Let $\alpha, p, q \in (0, 1]$ and suppose that, independently,

- a spreader involved in a meeting decides to tell the rumour with probability p;
- once such a decision is made, any spreader in a meeting with somebody informed has probability α of becoming a stifler;
- upon hearing the rumour, an ignorant becomes a spreader or a neutral with resp. probabilities q and $1 - q$.

Then, if $U(t)$ is the number of neutrals at time t, the CTMC $(X(t), U(t), Y(t))$ evolves according to

transition	rate
$(-1, 0, 1)$	$pq\, XY,$
$(-1, 1, 0)$	$p(1-q)\, XY,$
$(0, 0, -2)$	$\alpha^2 p(2-p)\binom{Y}{2},$
$(0, 0, -1)$	$\alpha(1-\alpha)p(2-p)\, Y(Y-1) + \alpha p\, Y(N+1-X-Y).$

The (α, p, q)-DK model

Let $\alpha, p, q \in (0, 1]$ and suppose that, independently,

- a spreader involved in a meeting decides to tell the rumour with probability p;
- once such a decision is made, any spreader in a meeting with somebody informed has probability α of becoming a stifler;
- upon hearing the rumour, an ignorant becomes a spreader or a neutral with resp. probabilities q and $1 - q$.

Then, if $U(t)$ is the number of neutrals at time t, the CTMC $(X(t), U(t), Y(t))$ evolves according to

transition	rate
$(-1, 0, 1)$	$pq\, XY$,
$(-1, 1, 0)$	$p(1-q)\, XY$,
$(0, 0, -2)$	$\alpha^2 p(2-p) \binom{Y}{2}$,
$(0, 0, -1)$	$\alpha(1-\alpha)p(2-p)\, Y(Y-1) + \alpha p\, Y(N+1-X-Y)$.

The (α, p, q)-DK model

Let $\alpha, p, q \in (0, 1]$ and suppose that, independently,

- a spreader involved in a meeting decides to tell the rumour with probability p;
- once such a decision is made, any spreader in a meeting with somebody informed has probability α of becoming a stifler;
- upon hearing the rumour, an ignorant becomes a spreader or a neutral with resp. probabilities q and $1 - q$.

Then, if $U(t)$ is the number of neutrals at time t, the CTMC $(X(t), U(t), Y(t))$ evolves according to

transition	rate
$(-1, 0, 1)$	$pq\, XY,$
$(-1, 1, 0)$	$p(1-q)\, XY,$
$(0, 0, -2)$	$\alpha^2 p(2-p)\binom{Y}{2},$
$(0, 0, -1)$	$\alpha(1-\alpha)p(2-p)\, Y(Y-1) + \alpha p\, Y(N+1-X-Y).$

The general model

- Let $V(t) = (X(t), U(t), Y(t))$.
- Initially, $X(0) = N$, $U(0) = 0$, $Y(0) = 1$ and $Z(0) = 0$, and $X(t) + U(t) + Y(t) + Z(t) = N + 1$ for all t.
- We suppose that $\{(X(t), U(t), Y(t))\}_{t \geq 0}$ is a CTMC with initial state $(N, 0, 1)$ and

transition	rate
$(-1, 0, 1)$	$\lambda\,\delta\,XY,$
$(-1, 1, 0)$	$\lambda(1-\delta)\,XY,$
$(0, 0, -2)$	$\lambda\,\theta_1 \binom{Y}{2},$
$(0, 0, -1)$	$\lambda\,\theta_2\,Y(Y-1) + \lambda\gamma\,Y(N+1-X-Y).$

The general model

- Let $V(t) = (X(t), U(t), Y(t))$.
- Initially, $X(0) = N$, $U(0) = 0$, $Y(0) = 1$ and $Z(0) = 0$, and $X(t) + U(t) + Y(t) + Z(t) = N + 1$ for all t.
- We suppose that $\{(X(t), U(t), Y(t))\}_{t \geq 0}$ is a CTMC with initial state $(N, 0, 1)$ and

transition	rate
$(-1, 0, 1)$	$\lambda \delta XY,$
$(-1, 1, 0)$	$\lambda(1-\delta) XY,$
$(0, 0, -2)$	$\lambda \theta_1 \binom{Y}{2},$
$(0, 0, -1)$	$\lambda \theta_2 Y(Y-1) + \lambda \gamma Y(N+1-X-Y).$

The general model

We define $\theta = \theta_1 + \theta_2 - \gamma$ and assume that

$$\lambda > 0,\ \gamma > 0,\ \theta_1 \geq 0,\ \theta_2 \geq 0,\ 0 < \delta \leq 1 \text{ and } 0 \leq \theta \leq 1. \qquad (1)$$

Remark

Stochastic rumour models reported in the literature:

λ	δ	γ	θ_1	θ_2	*Model*
1	1	1	1	0	*DK (1965)*
1	1	1	0	1	*MT (1973)*
p	1	α	$\alpha^2(2-p)$	$\alpha(1-\alpha)(2-p)$	(α, p)*-DK (1999)*
p	1	r/p	q_2/p	$q_1/(2p)$	*Pearce (2000)*
1	1	1	2	0	*Hayes (2005)*
$\tilde{\alpha}$	$\tilde{\theta}$	$\tilde{\gamma}/\tilde{\alpha}$	$\tilde{\beta}/\tilde{\alpha}$	0	*Kawachi (2008)*
p	q	α	$\alpha^2(2-p)$	$\alpha(1-\alpha)(2-p)$	(α, p, q)*-DK (2010)*

The general model

We define $\theta = \theta_1 + \theta_2 - \gamma$ and assume that

$$\lambda > 0,\ \gamma > 0,\ \theta_1 \geq 0,\ \theta_2 \geq 0,\ 0 < \delta \leq 1 \ \text{ and } \ 0 \leq \theta \leq 1. \qquad (1)$$

Remark

Stochastic rumour models reported in the literature:

λ	δ	γ	θ_1	θ_2	*Model*
1	1	1	1	0	*DK (1965)*
1	1	1	0	1	*MT (1973)*
p	1	α	$\alpha^2(2-p)$	$\alpha(1-\alpha)(2-p)$	(α, p)-*DK (1999)*
p	1	r/p	q_2/p	$q_1/(2p)$	*Pearce (2000)*
1	1	1	2	0	*Hayes (2005)*
$\tilde{\alpha}$	$\tilde{\theta}$	$\tilde{\gamma}/\tilde{\alpha}$	$\tilde{\beta}/\tilde{\alpha}$	0	*Kawachi (2008)*
p	q	α	$\alpha^2(2-p)$	$\alpha(1-\alpha)(2-p)$	(α, p, q)-*DK (2010)*

Some definitions

Definition

For $0 < \theta < 1$, *consider the function* $f_\theta : [0,1] \to \mathbb{R}$ *given by*

$$f_\theta(x) = \frac{(\gamma + \delta\theta)x^\theta - (\gamma + \delta)\theta x - \gamma(1-\theta)}{\theta(1-\theta)}.$$

For $\theta = 0$ *and* $\theta = 1$, *consider the functions* f_0 *and* f_1 *defined on* $(0,1]$ *by*

$$f_0(x) = (\gamma + \delta)(1-x) + \gamma \log x,$$

$$f_1(x) = -\gamma(1-x) - (\gamma + \delta)\, x \log x.$$

For each θ, *we define* $x_\infty = x_\infty(\delta, \gamma, \theta)$ *as the unique root of* f_θ *in the interval* $(0,1)$.

Our results: Law of large numbers

Theorem

Assume (1) and let x_∞ be as in last definition. Define

$$u_\infty = (1-\delta)(1-x_\infty).$$

Then,

$$\lim_{N\to\infty} \frac{X^{(N)}(\tau^{(N)})}{N} = x_\infty$$

and

$$\lim_{N\to\infty} \frac{U^{(N)}(\tau^{(N)})}{N} = u_\infty$$

in probability.

Our results: Central limit theorem

Theorem

We assume (1). Then,

$$\sqrt{N}\left(\frac{X^{(N)}(\tau^{(N)})}{N} - x_\infty, \frac{U^{(N)}(\tau^{(N)})}{N} - u_\infty\right) \xrightarrow{\mathcal{D}} N_2(0,\Sigma)$$

as $N \to \infty$, where $N_2(0,\Sigma)$ is the bivariate normal distribution with mean zero and covariance matrix Σ given by

$$\Sigma_{11} = x_\infty(1 - x_\infty) + A^2 C,$$
$$\Sigma_{12} = -(1-\delta)\Sigma_{11} + AB,$$
$$\Sigma_{22} = (1-\delta)^2\Sigma_{11} + (1-\delta)(\delta(1-x_\infty) - 2AB).$$

Remark

A, B and C can be computed as functions of our parameters. See Lebensztayn et al. (2010).

The proofs of the results rely on the theory of dependent Markov chains used by Kurtz et al. (2008) in the context of interacting random walks on the complete graph.

In brief, the stochastic process after a suitable acceleration converges to a deterministic system governed by a set of differential equations.

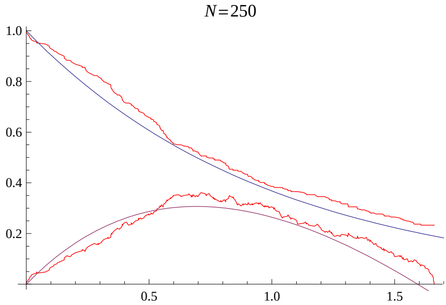

Figura: Behaviour of the MT model after acceleration.

The proofs of the results rely on the theory of dependent Markov chains used by Kurtz et al. (2008) in the context of interacting random walks on the complete graph.

In brief, the stochastic process after a suitable acceleration converges to a deterministic system governed by a set of differential equations.

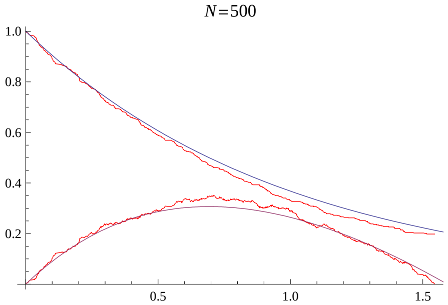

Figura: Behaviour of the MT model after acceleration.

Idea for the proofs

The proofs of the results rely on the theory of dependent Markov chains used by Kurtz et al. (2008) in the context of interacting random walks on the complete graph.

In brief, the stochastic process after a suitable acceleration converges to a deterministic system governed by a set of differential equations.

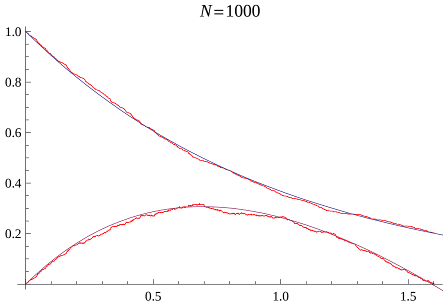

Figura: Behaviour of the MT model after acceleration.

Idea for the proofs

The proofs of the results rely on the theory of dependent Markov chains used by Kurtz et al. (2008) in the context of interacting random walks on the complete graph.

In brief, the stochastic process after a suitable acceleration converges to a deterministic system governed by a set of differential equations.

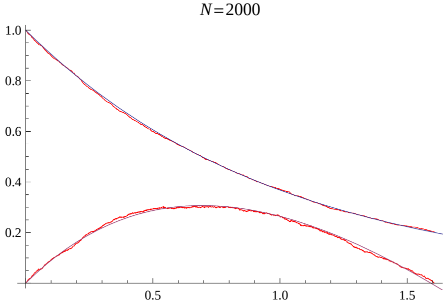

Figura: Behaviour of the MT model after acceleration.

Example

Let $\rho \in [0,1]$ *and consider our model with the choice* $\lambda = \delta = \gamma = 1$, $\theta_1 = \rho$ *and* $\theta_2 = 1 - \rho$, *so* $\theta = 0$.

Thus, the limiting proportion of ignorants and the variance of the asymptotic normal distribution in the CLT are given respectively by

$$x_\infty = x_\infty(1,1,0) = -\frac{W_0(-2\,e^{-2})}{2} \approx 0.203188, \quad \textit{and}$$

$$\sigma^2 = \frac{x_\infty(1-x_\infty)\left(1-2\,x_\infty+2\,\rho\,x_\infty^2\right)}{(1-2\,x_\infty)^2} \approx 0.272736 + 0.0379364\,\rho.$$

We obtain MT or DK model accordingly as ρ *equals* 0 *or* 1, *showing that our theorems generalize the results presented by Sudbury (1985) and Watson (1988).*

Example

Let $\rho \in [0,1]$ *and consider our model with the choice* $\lambda = \delta = \gamma = 1$, $\theta_1 = \rho$ *and* $\theta_2 = 1-\rho$, *so* $\theta = 0$.

Thus, the limiting proportion of ignorants and the variance of the asymptotic normal distribution in the CLT are given respectively by

$$x_\infty = x_\infty(1,1,0) = -\frac{W_0(-2\,e^{-2})}{2} \approx 0.203188, \quad \textit{and}$$

$$\sigma^2 = \frac{x_\infty(1-x_\infty)\left(1-2\,x_\infty+2\,\rho\,x_\infty^2\right)}{(1-2\,x_\infty)^2} \approx 0.272736 + 0.0379364\,\rho.$$

We obtain MT or DK model accordingly as ρ *equals* 0 *or* 1, *showing that our theorems generalize the results presented by Sudbury (1985) and Watson (1988).*

Example

Let $\rho \in [0,1]$ and consider our model with the choice $\lambda = \delta = \gamma = 1$, $\theta_1 = \rho$ and $\theta_2 = 1 - \rho$, so $\theta = 0$.

Thus, the limiting proportion of ignorants and the variance of the asymptotic normal distribution in the CLT are given respectively by

$$x_\infty = x_\infty(1,1,0) = -\frac{W_0(-2\,e^{-2})}{2} \approx 0.203188, \quad \textit{and}$$

$$\sigma^2 = \frac{x_\infty(1 - x_\infty)\left(1 - 2\,x_\infty + 2\,\rho\,x_\infty^2\right)}{(1 - 2\,x_\infty)^2} \approx 0.272736 + 0.0379364\,\rho.$$

We obtain MT or DK model accordingly as ρ equals 0 or 1, showing that our theorems generalize the results presented by Sudbury (1985) and Watson (1988).

Example

Hayes (2005) simulated the [MT] model, with the difference that, when two spreaders meet, both become stiflers. This model is obtained by choosing $\lambda = \delta = \gamma = 1$, $\theta_1 = 2$ and $\theta_2 = 0$, in which case $\theta = 1$,

$$x_\infty = x_\infty(1,1,1) = -\frac{1}{2\,W_{-1}(-e^{-1/2}/2)} \approx 0.284668, \quad \text{and}$$

$$\sigma^2 = \frac{x_\infty(1-x_\infty)\left(1-3\,x_\infty+3\,x_\infty^2\right)}{(1-2\,x_\infty)^2} \approx 0.427204.$$

This clarifies the numerical value of the proportion of the ignorants remaining in the population that Hayes obtained in his simulations.

Example

Hayes (2005) simulated the [MT] model, with the difference that, when two spreaders meet, both become stiflers. This model is obtained by choosing $\lambda = \delta = \gamma = 1$, $\theta_1 = 2$ *and* $\theta_2 = 0$, *in which case* $\theta = 1$,

$$x_\infty = x_\infty(1,1,1) = -\frac{1}{2\, W_{-1}(-e^{-1/2}/2)} \approx 0.284668, \quad \textit{and}$$

$$\sigma^2 = \frac{x_\infty(1 - x_\infty)\left(1 - 3\,x_\infty + 3\,x_\infty^2\right)}{(1 - 2\,x_\infty)^2} \approx 0.427204.$$

This clarifies the numerical value of the proportion of the ignorants remaining in the population that Hayes obtained in his simulations.

Examples

Example

Hayes (2005) simulated the [MT] model, with the difference that, when two spreaders meet, both become stiflers. This model is obtained by choosing $\lambda = \delta = \gamma = 1$, $\theta_1 = 2$ *and* $\theta_2 = 0$, *in which case* $\theta = 1$,

$$x_\infty = x_\infty(1,1,1) = -\frac{1}{2\,W_{-1}(-e^{-1/2}/2)} \approx 0.284668, \quad \textit{and}$$

$$\sigma^2 = \frac{x_\infty(1-x_\infty)\left(1-3\,x_\infty+3\,x_\infty^2\right)}{(1-2\,x_\infty)^2} \approx 0.427204.$$

This clarifies the numerical value of the proportion of the ignorants remaining in the population that Hayes obtained in his simulations.

Example

The (α, p, q)-DK model is obtained by the choice

$$\lambda = p,\ \delta = q,\ \theta_1 = \alpha^2(2-p),\ \theta_2 = \alpha(1-\alpha)(2-p),\ \ \textit{and}\ \gamma = \alpha$$

If $0 < \alpha(1-p) < 1$, then x_∞ is the unique root of the function

$$f^*(x) = \frac{(1+q(1-p))x^{\alpha(1-p)} - (\alpha+q)(1-p)x - 1 + \alpha(1-p)}{(1-p)(1-\alpha(1-p))}$$

in the interval $(0,1)$. When $q = 1$, this is exactly the limiting value obtained in the deterministic analysis of the model presented in Daley and Gani (1999).

Examples

Example

The (α, p, q)*-DK model is obtained by the choice*

$$\lambda = p,\ \delta = q,\ \theta_1 = \alpha^2(2-p),\ \theta_2 = \alpha(1-\alpha)(2-p),\ \ \text{and}\ \gamma = \alpha$$

If $0 < \alpha(1-p) < 1$, *then* x_∞ *is the unique root of the function*

$$f^*(x) = \frac{(1+q(1-p))x^{\alpha(1-p)} - (\alpha+q)(1-p)x - 1 + \alpha(1-p)}{(1-p)(1-\alpha(1-p))}$$

in the interval $(0, 1)$. *When* $q = 1$, *this is exactly the limiting value obtained in the deterministic analysis of the model presented in Daley and Gani (1999).*

Example

The (α, p, q)-DK model is obtained by the choice

$$\lambda = p,\ \delta = q,\ \theta_1 = \alpha^2(2-p),\ \theta_2 = \alpha(1-\alpha)(2-p), \ \ \textit{and} \ \ \gamma = \alpha$$

If $0 < \alpha(1-p) < 1$, *then* x_∞ *is the unique root of the function*

$$f^*(x) = \frac{(1+q(1-p))x^{\alpha(1-p)} - (\alpha+q)(1-p)x - 1 + \alpha(1-p)}{(1-p)(1-\alpha(1-p))}$$

in the interval $(0, 1)$. *When* $q = 1$, *this is exactly the limiting value obtained in the deterministic analysis of the model presented in Daley and Gani (1999).*

References

Daley, D.J. and Gani, J. (1999). Epidemic Modelling: an Introduction. Cambridge University Press, Cambridge.

Daley, D. J. and Kendall, D. G. (1965). Stochastic rumours. *J. Inst. Math. Appl.* **1** 42–55.

Hayes, B. (2005). Rumours and Errours. *American Scientist* **93** 207–211.

Kawachi, K. (2008). Deterministic models for rumor transmission. *Nonlinear Anal. Real World Appl.* **9** 1989–2028.

Kurtz, T. G.; Lebensztayn, E.; Leichsenring, A. and Machado, F. P. (2008). Limit theorems for an epidemic model on the complete graph. *Alea*, **4** 45–55.

Lebensztayn, E.; Machado, F.P. and Rodríguez. P.M. (2010). Limit theorems for a general stochastic rumour model. *arXiv: math.PR/1003.4995.*

Maki, D.P. and Thompson, M. (1973). Mathematical Models and Applications. Prentice-Hall, Englewood Cliffs.

Pearce, C. E. M. (2000). The exact solution of the general stochastic rumour. *Math. Comput. Modelling* **31** 289–298.

Sudbury, A. (1985) The proportion of the population never hearing a rumour. *J. Appl. Prob.* **22** 443–446.

Watson, R. (1988) On the size of a rumour. *Stoch. Proc. Appl.* **27** 141–149.